

Quantitative expression and aggregation of performance measurements based on the MACBETH multi-criteria method

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Abstract

This study deals with the use of the MACBETH methodology as a global framework for multi-criteria industrial performance expressions. This methodology satisfies the measurement theory requirements, and thus ensures the coherence of the elementary and aggregated performance expressions. The MACBETH procedure allows to express commensurate elementary performances and the weights of the weighted mean from decision-maker knowledge, and then to aggregate the elementary performances. Then, in order to take the criteria interactions into account, we extend MACBETH to the Choquet integral operators. Finally, an industrial case study is presented to illustrate the implementation of the methodology in the field.

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1. Introduction—problem statement

To deal with the complexity of the current industrial context, new strategies gaining wide acceptance when driving continuous improvement (Deming, 1982) generally include the following steps (Berrah et al., 2001; Bradford and Childe, 2001): identifying key areas, as-is situation analysis, planning and implementing changes, monitoring the results, and developing a closed-loop control system. To achieve all these functions, defining performance expressions and modelling their relationships are key activities. Indeed, according to their definition, the purpose of performance indicators is, on the one hand, to give pieces of information about the objective's satisfaction and, on the other hand, to link the current measures to the improvement actions to launch (Fortuin, 1988; Bitton, 1990; Berrah and Mauris, 2002). In this sense, the so-called performance measurement systems (PMS's) are the instruments to support decision-making, either for launching or choosing improvement actions or redefining objectives (Bititci, 1995; Bitton, 1990; Globerson, 1985; Neely, 1999).

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From a global point of view, a PMS can be seen as a multi-criteria instrument, made of a set of performance expressions (also referred to as “metrics” (Cooke, 2001; Melnyk et al., 2004), i.e. physical measures as well as performance evaluations, to be consistently organized with respect to the objectives of the company. Generally, the considered global objectives are broken down into elementary ones along organizational levels (strategic, tactical or operational). Thus, beyond the objective’s breakdown, one major problem in the design of PMS concerns the determination of performance expressions which are useful for the control decision-making (Ducq et al., 2001; Kainuma and Tawara, 2006). Two kinds of performance expressions are notably involved in a PMS, the elementary ones and the aggregated ones. The elementary expressions identify the reached degrees of the different objectives. The aggregated performance expressions are the synthesis of the elementary performance expressions into global ones.

Performance aggregation is defined as the corollary step of the objective breakdown. In the Taylorian organizations, aggregated expressions were not useful for control decision-making, elementary performances being independent, mono criterion and conventionally used only for a local verification. The weighted arithmetic mean (WAM) of the different performances simply gave some strategic information about the evolution of the global manufacturing productivity.

In the post-Taylorian context, the aggregation is essentially based on logical frameworks for linking strategic objectives, structuring the tactical and operational criteria affecting them. It generally deals with the translation of all the elementary performance expressions associated with the various heterogeneous criteria into a common reference (cost or satisfaction degree). The weighted mean, which is the more common compromise aggregation operator, is generally used to obtain the global performance. Most of the PMS’s echo this point of view (Globerson, 1985; Suwignjo and Bititci, 2000; Berrah et al., 2004). Indeed, the main frameworks used in the PMS literature are aimed at reducing the dimensionality, and thus are issued from the Multi Attribute Utility Theory (MAUT) (Dyer, 2005; Diakoulaki et al., 1992; Rangone, 1996; Bititci, 2001; Saaty, 2005; Kulak and Kahraman, 2005), knowing that there are limits to the decision-maker ability to process large sets of performance expressions. Note that a few studies are based on outranking models (Mareschal et al., 1991; Babic and Plazibat 1998; Brans and Mareschal, 2004). More precisely, in order to support the decision, the set of performance expressions has to be processed for comparing different situations that occur in the industrial context. Indeed, aggregation models enable to capture the notion of priorities in the decision-maker strategy (e.g. *investment*), and they allow the comparison of any two situations described through their elementary performance expressions.

Hence, in the current PMS’s, the quantification aspects (related to the parameters of the aggregation operator) are considered as an a posteriori problem, once performance expressions have been selected and defined. Nevertheless, the a posteriori definition of the aggregation model is no longer adapted to the current context, where criteria are dependent and interacting through the different levels of the company, and the control is reactive and global. Indeed, how can a coherent result be obtained if we elaborate elementary performance expressions independently of their further processing, namely the aggregation in this case? And as a corollary, how can an aggregation operator be defined without considering the performance structure? In this sense, with the current approaches, there is a lack of consistency between the elaboration of the elementary performance expressions and the aggregation one. In addition, the use of the weighted mean can also be restrictive knowing that the performance criteria are interacting. We have previously seen (Berrah et al., 2004; Globerson, 1985) that more generalized aggregation operators are relevant at the practical point of view, such as the Choquet integral (CI) aggregation operator (Grabisch, 1997; Berrah et al., 2003) that generalizes the weighted mean by taking mutual interactions between criteria into account.

Our propositions lie within the MAUT approach aggregation views (Figueira et al., 2004). And, in order to improve the coherence of the information processing involved in PMS’s and to provide useful performance expressions for control, our approach consists in considering the aggregation problem as a central structuring point. In this line, we propose to relate in a consistent and meaningful way the expressions of the elementary performances to the determination of the aggregation operator. This leads to consider two natural aspects:

- the commensurability of the elementary expressions, i.e. two identical performance values along two different criteria whatever they are must correspond to the same satisfaction degree for the decision-maker,

- the significance of the considered aggregation operator, i.e. the comparison (difference, ratio) of situations based on the aggregated expressions must be the same as the one based directly on the decision-maker observation.

In fact, the difficult problem of a sound quantification for the evaluations of observed situations is tackled by the measurement theory (Krantz et al., 1971). Thus, we consider the MACBETH (Measuring Attractiveness by a Categorical-Based Evaluation TecHnique) multi-criteria method (Bana e Costa and Vansnick, 1997) that satisfies the measurement theory requirements. This method defines quantitative performance expression and aggregation from qualitative pairwise comparisons of situations issued from the decision-maker. This is made in the same spirit as the AHP (Analytic Hierarchy Process) method (Saaty, 2005), but the results are processed according to the measurement theory. Moreover, as we will see later, the use of the MACBETH method offers many advantages for the industrial control. Particularly, it allows decision-makers to perform:

- continuous improvement, in which case the problem is to define for all the encountered situations the elementary and aggregated performance expressions,
- reengineering process, in which case the problem is to select the best action plans versus a limited number of action plans considered by the decision-maker w.r.t. the company strategy.

This paper is organized as follows. Section 2 focuses on the requirements about the industrial performance aggregation. The commensurability and the significance concepts are then introduced as a response to the coherence requirement between the elaboration of the elementary performances and their aggregation. Then, in Section 3, a global performance measurement framework is presented resulting from the MACBETH method. An extension of the latter to the 2-additive CI is also presented. Section 4 illustrates the proposed approach by means of a practical case study issued from a SME, that manufactures kitchens, bathrooms and storing spaces. The global objective of the company is to continuously increase the profit margins. In this sense, a strategic action plan, the “Quality and Environment Plan”, is defined, which includes the different actions and improvements to carry out on the manufacturing process and product quality on the one hand, and on the environment constraints on the other hand.

2. The industrial performance aggregation

2.1. Description of the problem

The aggregation of the performance expressions can be expressed as an operation that synthesizes the elementary performance expressions into a global performance expression. Hence, the performance aggregation can be formalized by the following mapping (Berrah et al., 2004):

$$\text{Ag} : E_1 \times E_2 \times \dots E_i \times \dots E_n \rightarrow E$$

$$(p_1, p_2, \dots, p_i, \dots, p_n) \rightarrow p_{\text{Ag}} = \text{Ag} (p_1, p_2, \dots, p_i, \dots, p_n)$$

The E_i 's are the universes of discourse of the elementary performance expressions $(p_1, p_2, \dots, p_i, \dots, p_n)$ and E is the universe of discourse of the global performance expression p_{Ag} . As the universes E_i 's and E can be different, the determination of the aggregation mapping Ag is, generally, not straightforward.

Let us consider a simple example with 3 criteria to illustrate this idea (an industrial application is presented Section 4). In order to compare the global performance of the different shop floors of the company, let us imagine that we need to aggregate the performances related to the three following criteria: the *manufacturing_productivity*, the *average_delay* and the *quality_rate*. For a given shop floor, the current states are given by the following measures: $m_1 = 1000$ pieces/hour for the *manufacturing_productivity*, $m_2 = 12$ h for the *average_delay* and $m_3 = 92.5\%$ for the *quality_rate*.

In this case, directly determining the shop floor global performance is a cumbersome task. Indeed, the elementary entities that are defined on different physical scales have to be related to a global entity, which is

defined on a satisfaction scale. One manner to simplify the previous problem is to transform the elementary physical performance measures into satisfaction degrees and to define the relation between the local satisfactions and the global one by a weighted mean.

Let us pursue the preceding example. In order to make the aggregation possible, one solution is to make a matching between measures and the assigned objectives, respectively noted m_i and o_i with $i = 1$ to n , respectively: $o_1 = 2000$ pieces/h; $o_2 = 0$ h $o_3 = 100\%$. One conventional way consists in computing ratios, as an extension of the Taylorian ones. These ratios are defined along the same universe $[0,1]$, where the value 1 means a total satisfaction of the objective while the value 0 means absolute non-satisfaction. Hence, for example,

- $p_1 = \frac{m_1}{o_1} = \frac{1000}{2000} = 0.50$ for manufacturing productivity,
- $p_2 = \frac{48-m_2}{48-o_2} = \frac{36}{48} = 0.75$, knowing that an *average_delay* of 48 h corresponds to absolute non-satisfaction,
- $p_3 = \frac{m_3-o_3}{o_3-0.9} = \frac{0.025}{0.10} = 0.25$, knowing that a *quality* rate of 90% corresponds to absolute non-satisfaction.

Besides, the aggregation being made thanks to the weighted mean, the decision-maker has to attribute weights to the different criteria $\sum_{i=1}^3 w_i = 1$, e.g. $w_1 = 0.5$, $w_2 = 0.25$, $w_3 = 0.25$. Hence,

$$p_{Ag} = w_1 p_1 + w_2 p_2 + w_3 p_3 = 0.50 \times 0.5 + 0.75 \times 0.25 + 0.25 \times 0.25 = 0.50. \quad (1)$$

This kind of approach is widely used in practice, but it raises many questions:

- how to define the transformation of physical measures into satisfaction degrees (by using ratio, difference, distance, etc.)?
- how to determine the aggregation parameters (i.e. the criteria weights in the case of the weighted mean operator)?

Obviously, the answer to the first question depends upon the objectives since the observed measures have to be compared with them. The answer to the second question depends upon the aggregation formulae. Indeed, the criteria weights in a weighted arithmetic mean and the criteria weights in a weighted geometric mean would not be the same. We will further discuss these aspects from an information processing point of view by using the measurement theory.

Lastly, note that the problem previously described is typical of the ones encountered at the tactical or strategic levels. At the operational level, the entities involved are often all defined on a physical scale and their relationship is also of a physical nature. For example, the *average_flow_time* of products for the whole manufacturing plant is a function of the *flow_times* of the different types of products, which themselves are a function of *flow_times* of their components. Though important (Neely, 1999), this latter aspect of defining a physical relation is not the object of this paper.

2.2. The elementary performance expression

Generally speaking, the transformation of physical measures into performance expressions can be given according to the following mapping (Berrah et al., 2000):

$$P : O \times M \rightarrow E,$$

$$(o, m) \rightarrow P(o, m) = p,$$

O , M and E are respectively, the universes of discourse of the set of objectives o , of the set of measures m and of the performance expression p . The key point in differentiating this kind of performance expression from conventional measurements is in the expression of a satisfaction degree (and not a physical measure), by the comparison of the acquired measures with an objective defined according to the control strategy considered. Thus, the mapping P denotes a comparison operator such as a distance operator or a similarity operator (Berrah et al., 2000).

This model is closely related to the metrics models (Melnyk et al., 2004) that recommend to consider for the universe E a set of values meaningful for decision-makers (monetary value, defect value, etc.). Here, for reasons of coherence of further information processing, we advocate a specific universe of satisfaction degree for E , i.e. the interval $[0,1]$ equipped with an adequate structure. More precisely, 0 means objective not satisfied at all and 1 means fully satisfied objective, and the higher the performance value is, the greater the satisfaction is (i.e. the performance is defined along an ordinal scale). Generally, a richer structure is desirable for control purposes, such as for example the meaningfulness of the difference of performances (i.e. the performance is defined along an interval scale).

2.3. The performance aggregation

In order to coherently realize the aggregation, the elementary performance values defined on the different intervals $[0,1]$ must:

- respect the commensurability requirement,
- be coherent with the chosen aggregation operator, generally the weighted mean.

The first point is referred to the commensurability issue that states that two identical values of the satisfaction degree must correspond to the same meaning for the decision-makers, whatever the criteria are. For example, a situation leading to a performance *manufacturing_productivity* of 0.7 and a situation leading to a performance *quality* of 0.7 are seen as having an equivalent satisfaction. Moreover, it can be useful, in addition to the commensurability, to have an interval scale for each elementary performance expression that requires an adequate procedure.

The second point means that the aggregation operator must be significant versus the structure of the elementary performances. For example, the aggregation operator being the weighted mean, a difference between two situations leading to performances *manufacturing_productivity* of 0.7 and 0.5 is seen as having an equivalent sense for the decision-makers as a difference between two situations leading to performances *manufacturing_productivity* of 0.5 and 0.3. In this case, an interval scale is obtained for the aggregated performance. Note also that it has been proved by (Labreuche and Grabisch, 2003) that if elementary performance expressions are defined on interval scales, the aggregated performance obtained by CI operators is also defined on an interval scale. In other respect, the aggregation operator parameters, e.g. weights of the weighted mean, have to be determined in relation to specific aggregated performance values in order to be determined in a significant way by the decision-makers.

2.4. Summary of the performance aggregation issue

The aggregation problem can thus be summarized by the following questions:

- how to build interval scales for commensurable elementary performance expressions?
- how to determine aggregation operator parameters, on the one hand, in a way consistent with the elaboration of the elementary performance expressions and, on the other hand, significant for the post-processing?

As it is generally difficult for decision-makers to directly answer these questions, adequate methods are required. For this reason, the MACBETH method presented hereafter proposes an interesting procedure to transform qualitative decision-maker's preferences into coherent (i.e. by satisfying the commensurability and the significance conditions) quantified elementary and aggregated performances (Clivillé, 2004).

3. Using MACBETH for coherent performance expression

3.1. Concepts and generalities

The Measuring Attractiveness by a Categorical-Based Evaluation TecHnique, MACBETH (Bana e Costa et al., 2005) is a MAUT method, which is based on the comparisons between different situations (which identify the context) made by the decision-makers. MACBETH describes these situations with, on the one hand, elementary performance expressions, and on the other hand, aggregated ones. In the MACBETH view, the definition of the aggregated performance is based on the weighted mean and is made progressively and interactively (Roubens et al., 2006). The principle is to translate the qualitative information generally available, thanks to the human expertise of the decision-makers, into quantitative information.

The elementary performance expressions are defined thanks to semantic judgments concerning the set of situations. More precisely, the concepts of preferences and strengths of preference are used, being linked to verbal levels of attractiveness (*null, very weak, weak, moderate, strong, very strong, extreme*) (Vansnick, 1984). The interval scales thus generated are in accordance with the MAUT and they are up to linear transformation (defined by two parameters: slope and constant) (Roubens, 1997). Then, to solve the inter-criteria commensurability issue, it is sufficient to determine, for all the interval scales, two common reference points having an absolute meaning for the decision-makers in order to set the slope and the constant. It is easy to consider, for example, “totally unsatisfying” associated to the performance value 0 and “totally satisfying” associated to the performance value 1.

Concerning the weights of the weighted mean operator, it is easy to see that the weight w_i corresponds to the global performance expression of the following elementary performance vectors $(0, \dots, 0, 1, 0, \dots, 0)$ where $p_i = 1$ and $p_j = 0$ for $j \neq i$. These vectors refer to characteristic situations, but as they are not ever observed, they also refer to fictive situations. Then, the procedure of weight quantification is also based on preferences and strengths of preference between the fictive situations in the same spirit as for the elementary performance determination. In this way, the weights are clearly related to the aggregation formulae (and not independently from it), which ensures the coherence of their determination.

Note that the MACBETH method only requires preference information about situations to evaluate. In fact, the latter which are generally defined through vectors of observed measures can be, in the case of the industrial control, associated to action plans. Indeed, decision-makers generally have many strategies. They are looking at performance expressions in order to follow the reached improvements. They may also want to determine the best action plan to implement in order to improve the performance. In both cases, the method provides a vector of elementary expressions noted $(p_1^k, \dots, p_i^k, \dots, p_n^k)$ where k characterizes the particular situation noted S^k and i characterizes the criterion noted C_i and an aggregated performance p_{Ag} (Bana e Costa et al., 2003).

3.2. The MACBETH procedure

The MACBETH procedure consists in four main steps (see Fig. 1). After the context definition (identification of the considered situations and criteria) (Step 1), MACBETH allows decision-makers to quantify in parallel:

- the vector of elementary expression (Step 2),
- the weights of the weighted arithmetic mean (Step 3).

Then, once the two types of information are available, MACBETH supplies the aggregated performance associated to the different situations (Step 4). Decision-makers can thus compare these expressions, determine the trend between past and current situations, identify the improvement key points for a given situation, etc. Note that a mechanism of verification of consistency of expert judgments is included in Steps 2 and 3 (see (Bana e Costa et al., 2005) for details).

Hereafter, the four mains steps are further detailed and illustrated with examples.

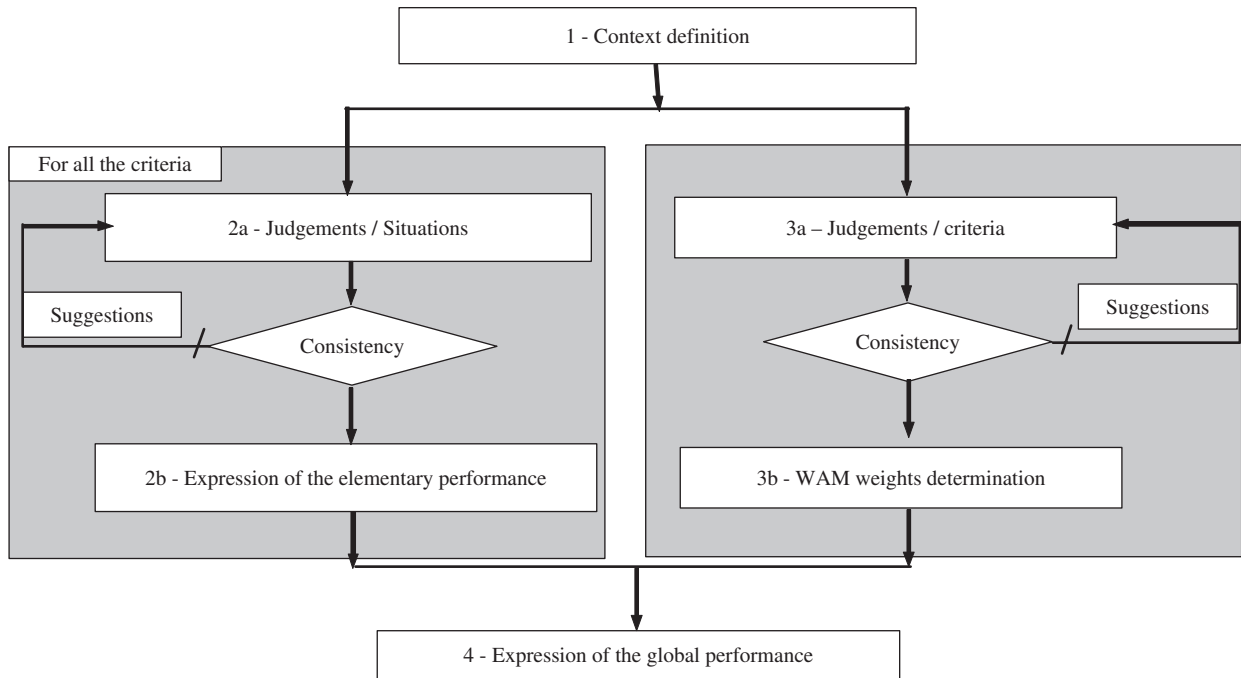


Fig. 1. MACBETH procedure for the performance expression.

3.2.1. The context definition step

The preliminary requirements for using the MACBETH methodology are to ask decision-makers about the evaluation problem. Thus, the essential elements concern first the identification of the different criteria according the global objective. Second, decision-makers are asked about the situations that have to be compared.

Example. Our example deals systematically with the case exposed Section 2. For the global objective $o_{\text{Man_prod}} = 2000$ pieces/h, the considered criteria are: $C_1 = C_{\text{Inv_level}}$, $C_2 = C_{\text{Tool_break}}$, $C_3 = C_{\text{Op_skill}}$. We can consider the set of situations observed at the end of each month of the first trimester.

$$S^1 : (m_1^1, m_2^1, m_3^1), \quad S^2 : (m_1^2, m_2^2, m_3^2), \quad S^3 : (m_1^3, m_2^3, m_3^3).$$

Note that in addition to the set of considered situations, two particular (possibly fictive) situations have to be introduced: the situation *good* is associated to an elementary performance expression of 1, the situation *neutral* is associated to an elementary performance expression of 0. Let us recall that these particular situations are reference points that allow to ensure commensurability between the different elementary performances.

3.2.2. The elementary performance expression step

The principle is to translate the human expertize concerning given situations into quantified elementary performance expressions, along an interval scale (Section 2.2). The expertize extraction is often a difficult problem that is easier to consider in two stages. The decision-makers are thus asked about, first their preferences, and second their strengths of preference (Section 3.1). According to the collected pieces of information, the elementary performances are then quantified.

3.2.2.1. The decision-maker's preferences. The preferences are provided for each considered criterion i . The information can be under the form:

- the decision-makers prefer for criterion i S^k to S^1 i.e. $S^k \succ S^1 \Leftrightarrow p_i^k > p_i^1$,
- S^k is equivalent for criterion i to S^1 i.e. $S^k \approx S^1 \Rightarrow p_i^k = p_i^1$.

Table 1

The preferences and strengths of preference of the decision-makers

Criterion: <i>Toolbreak</i>	Good	S^3	S^2	S^1	Neutral
Good	No	Very strong	Positive	Positive	Positive
S^3 : March		No	Moderate	Positive	Positive
S^2 : February			No	Moderate	Positive
S^1 : January				No	Weak
Neutral					No

The global result is then a ranked set of elementary expressions associated to each considered situation.

More particularly, as situations are known through their vectors of measures, noted $(m_1^k, \dots, m_i^k, \dots, m_n^k)$, for a given criterion i the information becomes:

- the decision-makers prefer for criterion i S^k to S^1 i.e. $S^k > S^1 \Leftrightarrow p_i^k > p_i^1$,
- m_i^k is equivalent to m_i^1 i.e. $m_i^k \approx m_i^1 \Leftrightarrow p_i^k = p_i^1$.

3.2.2.2. The decision-maker's strengths of preference. In the second stage, the decision-makers give more knowledge about the comparison between the considered situations. Indeed, they can characterize their preferences with a level of strength denoted by h . The h can take seven values, from 0 for a *null* strength, to 6 for an *extreme* strength. So, the previous information becomes: the decision-makers prefer S^k to S^1 with a strength h i.e. $S^k \succ^h S^1 \Leftrightarrow p^k - p^1 = h\alpha$, where α is a coefficient necessary to meet the condition p^k and $p^1 \in [0, 1]$.

More particularly, as situations are known through their vectors of measures, the information becomes: the decision-makers prefer m_i^k to m_i^1 with a strength h i.e. $m_i^k \succ^h m_i^1 \Leftrightarrow p_i^k - p_i^1 = h\alpha$, where α is the same coefficient as the one defined previously.

Example. Table 1 gives the decision-makers preferences and strengths of preferences w.r.t. the *toolbreak* criterion. For example, the decision-makers prefer moderately S^3 to S^2 . They simply prefer S^3 to S^1 .

3.2.2.3. The quantification of the elementary expressions. The quantification of the performance expressions is made by solving the equation system issued from the expression of all the strengths of preference h between S^k and S^1 , written under the form $p^k - p^1 = h\alpha$. For the sake of simplicity, we choose to illustrate the procedure through the considered example.

Example. Regarding the matrix given in Table 1, we have to solve the following equation system¹:

$$\begin{aligned}
 p^{\text{good}} - p^3 &= 1 - p^3 = 5\alpha, \\
 p^3 - p^2 &= 3\alpha, \\
 p^2 - p^1 &= 3\alpha, \\
 p^1 - p^{\text{neutral}} &= p^1 - 0 = 2\alpha,
 \end{aligned} \tag{2}$$

This system can be resolved by conventional techniques and it gives:

$$p^1 = \frac{2}{13} \approx 0.15, p^2 = \frac{5}{13} \approx 0.38, p^3 = \frac{8}{13} \approx 0.62 (\alpha = \frac{1}{13} \approx 0.08)$$

By this procedure, the elementary performance expressions are defined along interval scales defined on the interval $[0, 1]$ in a commensurate way.

Note that if we want to obtain the performance expressions for a lot of situations, the procedure becomes a cumbersome task. In this case, MACBETH proposes to make a linear interpolation between a limited number

¹To simplify the notation, we note $p_i^k : p^k$ knowing that the criterion i is the same for all the expressions.

of situations to quantify the performance associated to another measure m . In the simplest case, two main situations can be considered, *neutral* and *good*, through the associated measures m^{neutral} , m^{good} .

Example. The decision-makers define the *good* situation as the objective. They also know the measure corresponding to the *neutral* situation: $m_{\text{Tool_break}}^{\text{neutral}} = 10$ breaks/day.

Therefore, the performance expression will be by interpolation:

$$(p^{\text{neutral}} = 0, p^{\text{good}} = 1)p_{\text{Tool_break}} = \frac{m_{\text{Tool_break}} - m_{\text{Tool_break}}^{\text{neutral}}}{m_{\text{Tool_break}}^{\text{good}} - m_{\text{Tool_break}}^{\text{neutral}}} = \frac{-1}{9}m_{\text{Tool_break}}^{\text{neutral}} + \frac{10}{9}.$$

The measure $m_{\text{Tool_break}} = 3$ breaks/day corresponds to a performance $p_{\text{Tool_break}} = 7/9 \approx 0.78$. Moreover, if another situation is known, for example, $m_{\text{Tool_break}} = 5$ breaks/day which corresponds to a performance $p_{\text{Tool_break}} = 5/9 \approx 0.56$, it is naturally possible to improve the interpolation by defining a two piecewise linear function for the performance expression.

In fact, this procedure bridges the gap with the current practice that consists in directly determining an operator of comparison $p = f(o, m)$ between o and m . Therefore, if the knowledge of f is available for some criteria, it can directly be included in the procedure, the situation comparisons being avoided in this case. In this view, the MACBETH procedure is in the line of the frequent practice that consists in determining f directly; indeed it proposes a sound procedure to determine it in a piecewise linear form.

3.2.3. The WAM weights determination step

In order to determine the WAM weights, as for the elementary performance expression, MACBETH proposes to consider some particular situations and to compare them. Thus, MACBETH considers in priority characteristic situations (possibly fictive) in which are associated the elementary expression vectors such as $(0, \dots, 0, 1, 0, \dots, 0)$. Hence, the aggregated performance expression is reduced simply to: $p_{\text{Ag}}^i = w_i$ where p_{Ag}^i is the aggregated performance from the vector where $p_i = 1$ and all the other elementary expressions $p_g = 0$ with $g = 1$ to n and $g \neq i$.

To determine the n weights w_i 's (related to the n criteria involved), the decision-makers have to give n relations. In this order, MACBETH proposes a pairwise comparison between the characteristic situations. The result of each comparison is an equation, which can take the following form:

$$p_{\text{Ag}}^i - p_{\text{Ag}}^g = h\alpha = w_i - w_g \quad (3)$$

leading to a system of n independent equations.

Example. The characteristic situations are represented by their elementary performance expression vector $(p_{\text{Inv_level}}, p_{\text{Tool_break}}, p_{\text{Op_skill}})$. The decision-makers rank the characteristic situations then express their strengths of preference as follow:

$$S^{(0,0,1)} \succ S^{(1,0,0)} \succ S^{(0,1,0)} \succ S^{(0,0,0)},$$

$$S^{(0,0,1)} \succ_{\text{extrem}} S^{(1,0,0)} \succ_{\text{very_weak}} S^{(0,1,0)} \succ_{\text{weak}} S^{(0,0,0)}.$$

Considering the strength of preference h as integer, we can write the following system, where $w_1 = w_{\text{Inv_level}}, w_2 = w_{\text{Tool_break}}, w_3 = w_{\text{Op_skill}}$:

$$\begin{aligned} p_{\text{Ag}}^{(0,0,1)} - p_{\text{Ag}}^{(1,0,0)} &= 6\alpha = w_3 - w_1, \\ p_{\text{Ag}}^{(1,0,0)} - p_{\text{Ag}}^{(0,1,0)} &= \alpha = w_1 - w_2, \\ p_{\text{Ag}}^{(0,1,0)} - p_{\text{Ag}}^{(0,0,0)} &= 2\alpha = w_2, \\ w_1 + w_2 + w_3 &= 1. \end{aligned} \quad (4)$$

Table 2
Elementary and aggregated performance expressions

	p_{Ag}	p_1	p_2	p_3
Weights		0.21	0.14	0.64
S^3 : March	0.60	0.62	0.42	0.71
S^2 : February	0.63	0.38	0.28	0.79
S^1 : January	0.39	0.15	0.75	0.43

And we find the following solution:

$$w_1 = \frac{3}{14} \approx 0.21, \quad w_2 = \frac{2}{14} \approx 0.14, \\ w_3 = \frac{9}{14} \approx 0.64 \quad \left(\alpha = \frac{1}{14} \approx 0.07 \right).$$

3.2.4. The aggregation step

It is now possible to aggregate the elementary performance expressions, according to the situations S^k : $p_{ag}^k = w_1 \times p_1^k + \dots w_1 \times p_i^k + \dots w_n \times p_n^k$.

Example. The matrix given in Table 2 summarizes the elementary and aggregated performance expressions for the set of considered situations.

The m aggregated expressions p_{Ag}^k associated to the m situations S^k can be compared. Thus, as said before, decision-makers can:

- identify the trend of the aggregated performance in order to link the past events and the improvement actions with the obtained results,
- choose the best one if the maximal aggregated expression $\max(p_{Ag}^k)$ is satisfactory,
- diagnose the causes of the insufficient reached performance,
- define the action plans in order to increase the aggregated performance expression when $\max(p_{Ag}^k)$ is not sufficient.

Let us consider once again the previous example. Note that with the MACBETH procedure that leads to interval scales, the differences between the elementary and aggregated performance expressions have a sense (i.e. are meaningful), e.g. $p_{Ag}^3 - p_{Ag}^1 = 0.21$ and $p_{Ag}^2 - p_{Ag}^1 = 0.24$ can be compared. But nothing guaranties that the ratios of performance expressions have a sense.

3.2.5. Summary

This presentation of the MACBETH method shows that the qualitative expertize of the decision-maker can be used to quantify the elementary and aggregated performance expressions in a commensurate and significant way. Nevertheless, the use of the weighted mean aggregation operator is restrictive in the sense that it requires the independence of the criteria that sometimes interacts. In this view, in the following paragraph, we will thus present the extension of MACBETH to the CI operators that are more general than the weighted mean.

3.3. The extension of MACBETH to the 2-additive Choquet integral

3.3.1. The 2-additive Choquet integral

The operators of the CI family (Grabisch and Roubens, 1996) are interesting because they include a lot of generalized mean operators (i.e. those included between the *min* and the *max* operators). Moreover, they can be written under the form of a conventional weighted mean modified by effects coming from interactions between elementary performances. Finally, they are stable under linear transformation (Labreuche and

Grabisch, 2003) and thus consistent with the performance expressions on an interval scale on the universe $[0,1]$, which is quite natural for the criteria considered in industrial PMS's. This property constitutes the theoretical basis of the extension of the MACBETH procedure to the CI aggregation operators.

More precisely, we present briefly hereafter the 2-additive CI that considers only interactions in twos, and that is defined by two types of parameters (Grabisch, 1997):

- the weight of each elementary performance expression in relation to all the other contributions to the overall performance evaluation by the so-called Shapley parameters v_i 's, that satisfy the condition $\sum_{i=1}^n v_i = 1$, which is a natural condition for decision-makers;
- the interaction parameters I_{ij} of any pair of performance criteria, that range in $[-1,1]$:
 - a positive I_{ij} implies that the simultaneous satisfaction of objectives o_i and o_j is significant for the aggregated performance evaluation, but a unilateral satisfaction has no effect,
 - a negative I_{ij} implies that the satisfaction of either o_i and o_j is sufficient to have a significant effect on the aggregated performance evaluation,
 - a null I_{ij} implies that no interaction exists; thus v_i acts as the weights in a common WAM.

In the case of the performance expression, the aggregation formula by the 2-additive CI is given by:

$$p_{Ag} = \sum_{i=1}^n v_i p_i - \frac{1}{2} \sum_{i=1}^n I_{ij} |p_i - p_j|. \quad (5)$$

where $(p_1, \dots, p_i, \dots, p_n)$ is the vector of elementary expressions with the property that:

$$\left(v_i - \frac{1}{2} \sum_{j=1}^n |I_{ij}| \right) \geq 0 \quad \forall i \in [1, n] \text{ and } j \neq i. \quad (6)$$

Eq. (5) provides explanations to decision-makers on how the v_i and I_{ij} parameters influence the aggregated performance expressions.

3.3.2. Procedure for aggregated performance expressions by the CI

The procedure follows the same spirit as the one used for the WAM operator. The difference stands in the fact that more parameters have to be determined for the CI, namely, the *Shapley parameters* v_i 's and the interaction coefficients I_{ij} 's (Labreuche and Grabisch, 2003; Clivillé et al., 2004). Thus, the quantity of information to determine the parameters is larger, more situations have to be considered. The details of the procedure are described in the appendix and illustrated in the following case study.

4. Case study

4.1. Presentation

The case study concerns the Mobalpa company, a SME of 900 persons producing kitchens, bathrooms and storing spaces. The company manufactures more than 850 000 items its weekly production is 5000 pieces of furniture. The manufacturing process is moderately automated, consisting in 10 major activities (manufacturing, injection, assembly, finishing, packing, dispatching, etc.). The product is made of about 10 different parts, each part containing between 10 and 50 elementary pieces. The manufacturing cycle time is between 2 and 5 weeks.

The overall objective of the company is to continuously increase the profit margins. In this sense, the company defines an annual strategic action plan QEP "Quality and Environment Plan" which includes the different actions and improvements to carry out around the manufacturing process, the product quality and the environment constraints. More precisely, the top-management has defined the following five global objectives:

- the *service rate* must increase from 95% to 97.5%,
- the *after-sales service rate* must decrease from 1.35% to 1.2%,

- the product offer must remain very_satisfactory,
- the delivery delay must be reduced from 12 to 9 weeks,
- the social climate must become excellent.

Knowing that the company decides to improve its performance in terms of delay, let us consider more particularly the objective related to the *service_rate*. As stated earlier in this paper, the aim of this study is not to confine decision-makers to a narrow choice of performance indicators to assess and control their system, but to make a compromise among a limited set of relevant but conflicting performance evaluation criteria. Indeed, the company looks for the solution to the expression of the global performance related to the *service_rate* w.r.t. the elementary ones, knowing that the performance structure is implicitly known. This expression is notably useful for the comparison of the potential action plans of the company in this context. More particularly, the elementary performances are not systematically quantified, namely those associated to the action plans. Moreover, the interactions between the different criteria involved in the *service_rate* are not considered in the current practice. The approach suggested is therefore based on the MACBETH procedure (Section 3.2), with an extension to the 2-additive CI (Section 3.3).

4.2. The performance expression for the *service_rate*

4.2.1. Context definition

According to the MACBETH procedure, many parameters have to be preliminarily defined:

- the objectives related to the *service_rate*, as well as the situations good and neutral,
- the situations which have to be considered, namely those associated with the action plans.

Fig. 2 provides a partial decomposition of the strategic objective related to the *service_rate* into tactical objectives and basic criteria to be used to assess operational objectives.

For the sake of conciseness, we consider here the strategic and tactical levels, the handling of the operational level being similar. The retained criteria for the *service_rate* are thus: the *average_delay*, the *order_receive_time* and the *accuracy_shipments*.

Hence, the *good* situation corresponds to the complete satisfaction of the objectives. The *neutral* situation corresponds to the no satisfaction of the objectives. In our case, this latter situation corresponds to the current ones observed before conducting improvement actions (Table 3).

Besides, in accordance with the quality manager, the team management generates several action plans (Table 4). The set of considered situations, associated to generated action plans are noted $\{S_{CI}; S^{HR}; S^{TI}; S^{QE}\}$.

Let us recall that knowing that all these action plans cannot be simultaneously launched w.r.t. the financial constraints of the company, the problem is thus to identify those which improve the global performance related to the *service_rate* the most significantly.

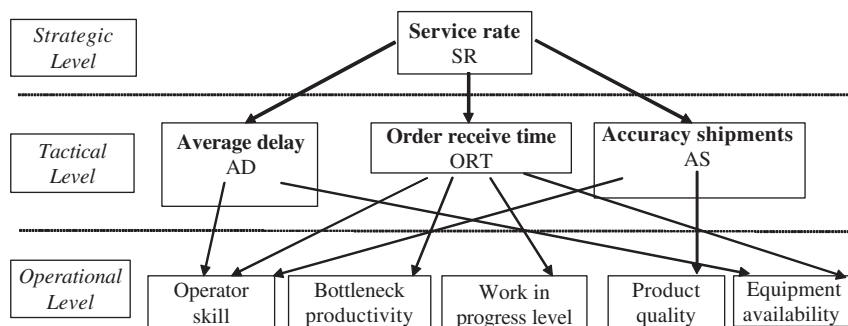


Fig. 2. The tree structure of the *service_rate* global objective.

Table 3

The particular situations “good” and “neutral” for the service_rate

Criterion	Decisional level	Good situations (objectives)	Neutral situations (current measures)
Service rate	Strategic	$o_{sr} = 97.5\%$	$m_{SR} = 90\%$
Average delay	Tactical	$o_{ad} = 1 \text{ day}$	$m_{AD} = 5 \text{ days}$
Time between order and receive	Tactical	$o_{ort} = 6 \text{ weeks}$	$m_{ORT} = 12 \text{ weeks}$
Accuracy of shipments	Tactical	$o_{as} = 99\%$	$m_{AS} = 90\%$

Table 4

The potential action plans of the company

Service	Action plan label and associated situation
Quality and environment	Critical assessment of the power and equipment used: S^{QE}
Human resources	Training plan for the operators: S^{HR}
Technical investment	Purchase of new assembly units: S^{TI}
Continuous improvement	Autonomous team development and systematic performance indicators using: S^{CI}

4.2.2. The elementary performance expressions of the action plans

The quality manager is asked about his preferences and strengths of preference, for a given criterion, w.r.t. the different action plans.

Preferences:

$$\begin{aligned}
 \text{Average_delay:} & \quad S^{HR} \succ S^{CI} \succ S^{QE} \succ S^{TI}, \\
 \text{Order_receive_time:} & \quad S^{HR} \succ S^{CI} \succ S^{QE} \succ S^{TI}, \\
 \text{Shipment_accuracy_delay:} & \quad S^{TI} \succ S^{QE} \succ S^{CI} \succ S^{HR}.
 \end{aligned}$$

Strengths of preference:

$$\begin{aligned}
 \text{Average_delay:} & \quad \text{good} \succ^2 S^{HR} \succ^1 S^{CI} \succ^4 S^{QE} \succ^2 S^{TI} \succ^6 \text{neutral}, \\
 \text{Order_receive_time:} & \quad \text{good} \succ^1 S^{HR} \succ^2 S^{CI} \succ^1 S^{QE} \succ^3 S^{TI} \succ^4 \text{neutral}, \\
 \text{Shipment_accuracy_delay:} & \quad \text{good} \succ^2 S^{TI} \succ^2 S^{QE} \succ^1 S^{CI} \succ^5 S^{HR} \succ^2 \text{neutral}.
 \end{aligned}$$

An illustration, Fig. 3, issued from the MACBETH software² shows the elementary performance expression related to the *average_delay*. The performance scale results from a resolution of the equation system based on the table of preferences, strengths of preference (see Section 3.2).

The other expressions are found by the resolution of the respective equation systems according to the method presented in Section 3.2.

$$\begin{aligned}
 \text{Average_delay:} & \quad p_{AD}^{HR} \approx 0.87, \quad p_{AD}^{CI} = 0.80, \quad p_{AD}^{QE} \approx 0.53, \quad p_{AD}^{TI} \approx 0.47, \\
 \text{Order_receive_time:} & \quad p_{ORT}^{HR} \approx 0.91, \quad p_{ORT}^{CI} \approx 0.73, \quad p_{ORT}^{QE} \approx 0.64, \quad p_{ORT}^{TI} \approx 0.36, \\
 \text{Shipment_accuracy_delay:} & \quad p_{AS}^{TI} \approx 0.83, \quad p_{AS}^{QE} \approx 0.67, \quad p_{AS}^{CI} \approx 0.58, \quad p_{AS}^{HR} \approx 0.17.
 \end{aligned}$$

4.2.3. The CI parameter determination

According to the interactions between the considered criteria, we use the 2-additive CI operator for the performance aggregation. We have then to determine 3 Shapley parameters $v_1 = v_{AD}$, $v_2 = v_{ORT}$, $v_3 = v_{AS}$ and 3 interaction coefficients I_{12} , I_{13} , I_{23} .

²A free version is available at <http://www.m-macbeth.com/>.

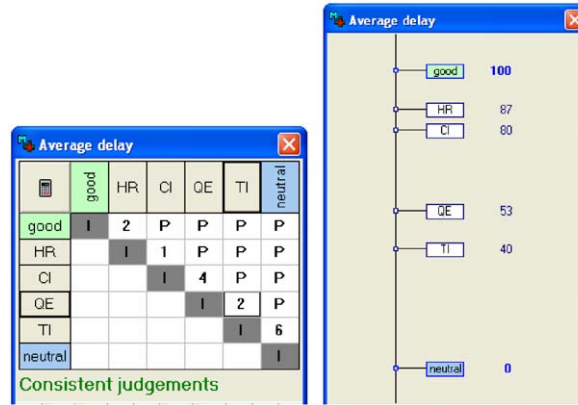


Fig. 3. The average_delay preferences, strengths of preference and performance scale.

The quality manager has to supply information about 6 characteristic situations that are fictive in this case. The associated performance vectors are, respectively, $(1,0,0)$, $(0,1,0)$, $(0,0,1)$, $(0,1,1)$, $(1,0,1)$, $(1,1,0)$. According to the MACBETH procedure, the quality manager expresses first his preferences concerning these situations, then he provides more knowledge with his strengths of preference. We note, for example, $p_{Ag}^{(0,1,1)}$ the aggregate performance associated to the situation described by the vector $(0,1,1)$.

Preferences:

$$p_{Ag}^{(0,1,1)} > p_{Ag}^{(1,0,1)} > p_{Ag}^{(1,1,0)} \approx p_{Ag}^{(0,0,1)} > p_{Ag}^{(0,1,0)} > p_{Ag}^{(1,0,0)} > p_{Ag}^{(0,0,0)}.$$

Strengths of preference:

$$p_{Ag}^{(0,1,1)} >^3 p_{Ag}^{(1,0,1)} >^6 p_{Ag}^{(1,1,0)} >^0 p_{Ag}^{(0,0,1)} >^2 p_{Ag}^{(0,1,0)} >^1 p_{Ag}^{(1,0,0)} >^2 p_{Ag}^{(0,0,0)}.$$

These pieces of information lead to write the following equation system:

$$p_{Ag}^{(0,1,1)} - p_{Ag}^{(1,0,1)} = 3\alpha = v_2 - v_1 - 0.5(I_{13} - I_{23}),$$

$$p_{Ag}^{(1,0,1)} - p_{Ag}^{(1,1,0)} = 6\alpha = v_3 - v_2 - 0.5(I_{12} - I_{13}),$$

$$p_{Ag}^{(1,1,0)} - p_{Ag}^{(0,0,1)} = 0 = v_1 + v_2 - v_3,$$

$$p_{Ag}^{(0,0,1)} - p_{Ag}^{(0,1,0)} = 2\alpha = v_3 - v_2 - 0.5(I_{13} - I_{12}),$$

$$p_{Ag}^{(0,1,0)} - p_{Ag}^{(1,0,0)} = \alpha = v_2 - v_1 - 0.5(I_{23} - I_{13}),$$

$$p_{Ag}^{(1,0,0)} - p_{Ag}^{(0,0,0)} = 2\alpha = v_1 - 0.5(I_{12} + I_{13}),$$

$$v_1 + v_2 + v_3 = 1.$$

(7)

The resolution of the system gives

$$v_1 = v_{AD} = 0.2, v_2 = v_{ORT} = 0.3, v_3 = v_{AS} = 0.5,$$

$$I_{12} = 0, I_{13} = 0.2, I_{23} = 0.3 (\alpha = 0.1).$$

Table 5 summarizes the aggregated performances related to the considered fictive situations. These pieces of information help the quality manager to agree (or not) with the computed parameters. For instance, considering the vector $(1,0,1)$, he asks himself if the aggregated performance really corresponds to his expertise. If he does not agree, he has to modify its strengths of preference in order to best translate his knowledge.

4.2.4. The service_rate performance aggregation

The elementary expressions and the parameters of the CI being defined (Section 4.3.3), the aggregated performances can be computed (Table 6).

Table 5
Elementary and aggregated performance expression for the fictive situations

	p_1	p_2	p_3	p_{Ag}
$S^{(0,0,0)}$	0	0	0	0
$S^{(1,0,0)}$	1	0	0	0.10
$S^{(0,1,0)}$	0	1	0	0.10
$S^{(0,0,1)}$	0	0	1	0.25
$S^{(1,1,0)}$	1	1	0	0.25
$S^{(1,0,1)}$	1	0	1	0.55
$S^{(0,1,1)}$	0	1	1	0.70
$S^{(1,1,1)}$	1	1	1	1

Table 6
The case study elementary and aggregated performance expressions

	p_{Ag}	p_{AD}	p_{ORT}	p_{AS}	Part of interaction
S^{HR}	0.50	0.87	0.91	0.17	0.18
S^{OE}	0.61	0.53	0.64	0.67	0.02
S^{TI}	0.51	0.47	0.36	0.83	0.11
S^{CI}	0.63	0.80	0.73	0.58	0.06

We can see that interaction influences the aggregated performance for the action plan S^{HR} by a value of 0.18, that represents around 40% of the performance p_{Ag}^{HR} . On the opposite, interactions are negligible (0.02) for the aggregated expression p_{Ag}^{QE} because the elementary expressions are very close to each other. In the case of the considered action plans, taking the interactions into account gives an important difference between the aggregated performances p_{Ag}^{HR} and p_{Ag}^{CI} (0.50 and 0.63) which otherwise should be null (0.68 and 0.69).

As a conclusion, the quality manager deduces from the MACBETH procedure that they are two action plans which ensure a real improvement. Nevertheless, there is no action plan, which allows the company to reach completely the global objective related to the *service_rate*, the best aggregated expression being only of 0.63. Thus, the set of action plans probably has to be modified. In this order, the initial solution (S^{CI} : continuous improvement) can be compared to a the new one ($S^{CI_evolution}$: continuous improvement evolution). This new solution has, in particular, to improve the elementary performance p_{AS} .

More generally, the quality manager can now simulate the impact of different action plans, and consider the contradiction, the redundancy or the synergy between them. Then, its quantification makes his choices stricter. So he can carry out the quality and environment Plan thanks to this methodology, which avoids inefficient actions and identifies the shortcomings concerning some strategic objectives.

5. Conclusion

The analysis presented in this article is in keeping with the evolution of the multi-criteria industrial performance expression issue. The benefits from performance expressions at all the decision levels in order to improve control responsiveness are today acknowledged as a matter of fact. But the question of a sound processing of the set of these performances is still open. In this order, we have proposed a characterization of this problem from the angle of the aggregation process, which allows the manager to synthesize the performance from the vector of elementary expressions. Thus, we have considered the problem to express coherently the performance all over the company, particularly from the elementary ones to the global one. The coherence requirement has been assimilated to the commensurability and the significance concepts issued from the measurement theory.

In this sense, at the information processing point of view, we have considered the MACBETH multi-criteria method to quantify the elementary performances and the aggregated one in a coherent way from qualitative

knowledge given by decision-makers. An extension of the procedure to the 2-additive Choquet integral has been done, as this operator is well suited to deal with the interactions between the performance criteria. An industrial application has allowed us, on the one hand, to show the relevance of such an approach, on the second hand, to evaluate the knowledge required to carry out this type of method. The implementation of our PMS propositions in the information system of the company is in progress.

The main perspectives of this work will concern the definition of performance diagnosis functionalities. Indeed, to support the manager control, links between unsatisfactory global performances and the elementary ones have to be exhibited.

Appendix

This appendix details the Section 3.3.2.

In order to determine the 2 additive CI parameters, we propose to consider:

- as for the procedure to determine the WAM weights, situations where one elementary expression p_i is good, $p_i = 1$ and all the other elementary expressions p_j are bad $p_j = 0$. As we have seen before, this situations can be described by elementary expression vectors such as $(0, \dots, 0, 1, 0, \dots, 0)$. In this case, the aggregated performance expression becomes:

$$p_{Ag}^i = v_i - \frac{1}{2} \sum_{j=1}^n I_{ij}. \quad (8)$$

- in addition, situations where one elementary expression p_i is bad, $p_i = 0$ and all the other elementary expressions p_j are good, $p_j = 1$, these situations can be described by elementary expression vectors such as $(1, \dots, 1, 0, 1, \dots, 1)$. In this case, the aggregated performance expression becomes:

$$\begin{aligned} p_{Ag}^{\bar{i}} &= v_1 + \dots v_{i-1} + v_{i+1} + \dots \\ v_n - \frac{1}{2} \sum_{j=1}^n I_{ij} &= 1 - v_i - \frac{1}{2} \sum_{j=1}^n I_{ij}. \end{aligned} \quad (9)$$

In order to determine the *Shapley parameters* v_i 's and the interaction coefficients I'_{ij} 's, the decision-makers have to express enough aggregated expressions, i.e. as many as the sum of the number of *Shapley parameters* and the number of interaction coefficients.

Example. The considered situations are represented by their elementary performance expression vectors $(p_{Inv_level}, p_{tool_break}, p_{Op_skillfulness})$. The decision-makers rank the characteristic situations as follows:

$$S^{(1,0,1)} \succ S^{(0,1,1)} \succ S^{(0,0,1)} \succ S^{(1,1,0)} \succ S^{(1,0,0)} \succ S^{(0,1,0)} \succ S^{(0,0,0)}.$$

Preferences and strengths of preference are summarized in the matrix given in Table 7.

Considering the strength of preference h as an integer, we can write the following system, where $v_1 = v_{Inv_level}$, $v_2 = v_{Tool_break}$, etc.

$$\begin{aligned} p_{Ag}^{(1,0,1)} - p_{Ag}^{(0,1,1)} &= \alpha = v_1 - v_2 - 0.5(I_{23} - I_{13}), \\ p_{Ag}^{(0,1,1)} - p_{Ag}^{(0,0,1)} &= 2\alpha = v_2 - 0.5(I_{12} - I_{23}), \\ p_{Ag}^{(0,0,1)} - p_{Ag}^{(1,1,0)} &= 3\alpha = -1 + 2v_3, \\ p_{Ag}^{(1,1,0)} - p_{Ag}^{(1,0,0)} &= 3\alpha = v_2 - 0.5(I_{23} - I_{12}), \\ p_{Ag}^{(1,0,0)} - p_{Ag}^{(0,1,0)} &= \alpha = v_1 - v_2 - 0.5(I_{13} - I_{23}), \end{aligned}$$

Table 7
The CI preferences and strengths of preference between the fictive situations

	$S^{(1,0,1)}$	$S^{(0,1,1)}$	$S^{(0,0,1)}$	$S^{(1,1,0)}$	$S^{(1,0,0)}$	$S^{(0,1,0)}$	$S^{(0,0,0)}$
$S^{(1,0,1)}$	No	Strong	Positive	Positive	Positive	Positive	Positive
$S^{(0,1,1)}$		No	V. strong	Positive	Positive	Positive	Positive
$S^{(0,0,1)}$			No	Extreme	Positive	Positive	Positive
$S^{(1,1,0)}$				No	Moderate	Positive	Positive
$S^{(1,0,0)}$					No	Weak	Positive
$S^{(0,1,0)}$						No	Strong
$S^{(0,0,0)}$							No

Table 8
The CI elementary and aggregated performance expressions

	p_{AG} Choquet integral	p_1	p_2	p_3	Recall of p_{AG} WAM
S^3 : March	0.60	0.62	0.42	0.67	0.60
S^2 : February	0.59	0.38	0.28	0.80	0.63
S^1 : January	0.38	0.15	0.75	0.40	0.39

$$p_{Ag}^{(0,1,0)} - p_{Ag}^{(0,0,0)} = 2\alpha = v_2 - 0.5(I_{12} + I_{23}),$$

$$v_1 + v_2 + v_3 = 1 \quad (10)$$

The resolution of this system gives the following solution:

$$v_1 \approx 0.25 \quad v_2 \approx 0.014 \quad v_3 \approx 0.61,$$

$$I_{12} = -0.04, \quad I_{13} = 0.11, \quad I_{23} = 0.04 \quad \alpha \approx 0.07.$$

Once the different parameters are defined, it is possible to express the aggregated performance according to the set of situations S^k :

$$p_{Ag}^k = v_1 \times p_1^k + \dots v_i \times p_i^k + \dots v_n \times p_n^k - \frac{1}{2} \sum_{j=1}^n I_{ij} \times |p_i^k - p_j^k|. \quad (11)$$

Example. It is now possible to calculate the aggregated expression thanks to the following formula:

$$p_{Ag}^k = 0.25p_1^k + 0.14p_2^k + 0.61p_3^k - 0.5[-0.04|p_1^k - p_1^k| + 0.04|p_2^k - p_3^k| + 0.11|p_1^k - p_3^k|]. \quad (12)$$

The results are presented Table 8.

The conclusion is modified versus the one in Section 3.2.4. Indeed, the aggregated performance expression for February is now slightly better than the one for March. Therefore, the consequences of the interaction coefficients can modify the actions the decision-makers would launch to improve the situation.

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